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A SEQUENTIAL k-GROUP RANDOM ALLOCATION METHOD WITH APPLICATIONS TO SIMULATION

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ABSTRACT

A sequential method of random allocation is given and it is shown how it can be used to estimate the observed significance levels of k-sample nonparametric tests. The sequential technique is compared to the standard random allocation technique and shown to be more efficient. An application is made to the Dunn-Bonferroni method of multiple comparisons.

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Key Words: Dunn-Bonferroni method; Nonparametric tests; Observed significance levels; Simulation

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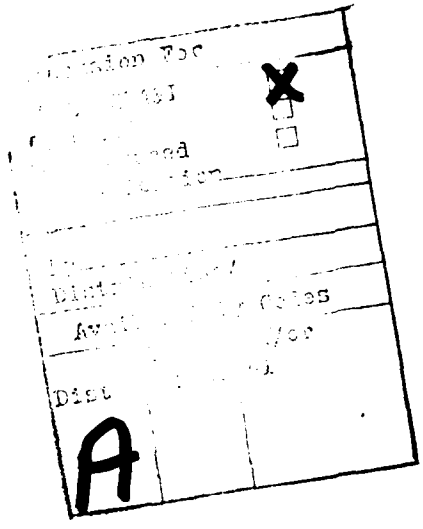
SIGNIFICANCE AND EXPLANATION

A major problem in using nonparametric tests is to determine the observed level of significance. This paper gives an efficient sequential simulation method for doing this.



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A SEQUENTIAL k-GROUP RANDOM ALLOCATION METHOD WITH APPLICATIONS TO SIMULATION

Andrew P. Soms

1. The Sequential Allocation Method

Bebbington (1975) showed that if there were N objects (such as file cards) from which it was desired to select (without replacement here and throughout) a random sample of size k without numbering the N objects, then one could proceed sequentially by selecting the first object with probability k/N and if at the $T^{\underline{th}}$ stage s have been selected, then the $T+1^{\underline{st}}$ object is selected with probability $(k-s)/(N-T)$, $T = 1,2,\ldots,N-1$.

We now state and prove the extension to an arbitrary number of groups. Suppose there are N objects and it is desired to sequentially divide them randomly into r groups of size $k_1,k_2,\ldots,k_r$, $\sum_{i=1}^{r} k_i = N$, i.e., each allocation has probability $1/\binom{N}{k_1,\ldots,k_r}$. Let $s_{1T},\ldots,s_{rT}$ be the number of objects selected for groups $1,2,\ldots,r$ at the $T^{\underline{th}}$ stage and let $P_{i,T+1}$ denote the selection probability for group i at the $T+1^{\underline{st}}$ stage. Then if

$$P_{i,T+1} = (k_i - s_{iT})/(N-T), \quad T = 0,1,\ldots,N-1 , \qquad (1.1)$$

the selection is random. Note that $P_{i,1} = k_i/N$ and $\sum_{i=1}^{r} P_{i,T+1} = 1$. The randomness follows immediately by noting that the probability of a particular assignment is

$$\left(\prod_{i=1}^{r} k_i!\right)/N! = 1/\binom{N}{k_1,\ldots,k_r} .$$

Bebbington's (1975) result is a special case of the above when r = 2.

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As an example, suppose $r = 3$, $k_1 = 2$, $k_2 = 2$, $k_3 = 3$ and $N = 7$. In order to make the sequential allocation given by (1.1) we take 7 independent random numbers U_i, $i = 1,2,\ldots,7$. Let

$$Q_{0,T} = 0 \text{ and } Q_{i,T} = \sum_{j=1}^{i} P_{j,T}, \quad i = 1,2,\ldots,r, \quad T = 1,2,\ldots,N.$$

Then the m^{th} object, $m = 1,2,\ldots,N$, is assigned to group n, where n is the unique integer such that

$$Q_{n-1,m} < U_m \leq Q_{n,m}.$$

Suppose the 7 random numbers are .79039, .01850, .99744, .81812, .93169, .22705, and .97709. The selection process is summarized in Table 1.

1. Selection Process

Stage	Random Digit	P_{1T}	P_{2T}	P_{3T}	Group Selected
1	.79039	2/7	2/7	3/7	3
2	.01850	2/6	2/6	2/6	1
3	.99744	1/5	2/5	2/5	3
4	.81812	1/4	2/4	1/4	3
5	.93169	1/3	2/3	0	2
6	.22705	1/2	1/2	0	1
7	.97709	0	1	0	2

Note that if all the k_i's are one, a random permutation is produced if we think of the group as denoting position.

2. Applications to Simulation

In k-sample nonparametric tests the observed significance level of the test is obtained by considering all possible partitions M of the (possibly tied) observed values or (possibly average) ranks into r groups, computing the value of the test statistic, and counting the number of times m it is equal to or greater than the observed value. The observed significance level $\hat{\alpha}$ is then m/M. When the number of partitions is large this is prohibitive and $\hat{\alpha}$ is estimated either by simulation (taking a large random sample of the allocations) or by asymptotics. The advantage of simulation is that one can control the accuracy of the estimate (by taking a large or small random sample) depending on the importance of the situation, unlike asymptotics which each time it is used forces one into the straight-jacket of committing a usually unknown error. Since it is (perhaps regrettably) a well known fact that different actions will be taken for close values of $\hat{\alpha}$, one above and the other below some fixed level (e.g., .01, .05, or .1) of the decision-maker, the use of simulation at least prevents approximating error in $\hat{\alpha}$ to be the determining factor.

If it is decided to use simulation, then a possible procedure is to make the random assignment as described in Section 1 many times by using a computer. The commonly used method is to produce a random permutation by ordering a random sample of uniform numbers and choosing the first k_1 indexes for group 1, the next k_2 for group 2, and so on. If all the k_i's are one, then this is more efficient than Section 1. However, as soon as the k_i's

depart even moderately from 1, the method of Section 1 becomes much more efficient. As an example, if $k_1 = k_2 = k_3 = k_4 = 10$ and it is desired to make 2000 random assignments using a UNIVAC 1110 computer, a FORTRAN program using the methods of Section 1 uses 4.71 seconds of CPU time while a FORTRAN program using the random permutation method takes 9.17 seconds.

The Appendix contains a listing of the FORTRAN subroutine RANDM that uses the theory of Section 1 to make random assignments. This may be tied in with any specific simulation problem, e.g., the case treated in Section 3.

3. Applications to the Dunn-Bonferroni Method of Multiple Comparisons

The D-B (Dunn-Bonferroni) method is described in Dunn (1964). Briefly, let Y_{ij}, $i = 1,2,\ldots,r$, $j = 1,2,\ldots,n_i$, be continuous (this assumption is not important and is removed later) random variables with distribution function F_i, H_0: $F_1 = F_2 = \ldots = F_r$, H_a: for at least one pair (i,j), $F_i \neq F_j$ (in the sense of producing larger or smaller values), and the test must identify which, if any, pairs are different. Denote by z_α the upper $\alpha^{\underline{th}}$ point of the standard normal. The D-B test declares all those pairs (i,j), $i < j$, different for which

$$z_{ij} = |\bar{R}_i - \bar{R}_j| \Big/ \left[\frac{(N)(N+1)}{12}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)\right]^{1/2} \geq z_{\alpha/(k(k-1))} \quad , \quad (3.1)$$

where $\bar{R}_i$ denotes the average of the ranks of the $i^{\underline{th}}$ group in the joint ranking. The nominal significance level of this procedure is α. The actual significance level α_A is

$$\alpha_A = P_0\left[\max_{i<j} z_{ij} \geq z_{\alpha/(k(k-1))}\right] \quad , \quad (3.2)$$

and may be obtained by simulation based on Section 1. Table 2 gives some comparisons of nominal with actual, using Section 1 and 10,000 simulations.

2. Comparison of Actual to Nominal α

r	Common Group Size	Nominal α	Actual α
3	5	.05	.037
3	10	.05	.040
3	15	.05	.043
3	30	.05	.045
3	5	.01	.0030
3	10	.01	.0077
3	15	.01	.0077
3	30	.01	.010
5	5	.05	.026
5	10	.05	.036
5	5	.01	.0030
5	10	.01	.0067

The Appendix contains a listing of the program used for Table 2. It thus appears that D-B is conservative and we can remove the conservatism by substituting for $z_\alpha/(k(k-1))\ d_{(i)}$, where $d_{(i)}$, $i = 1,2,\ldots,r(r-1)/2$, is the $i^{\underline{th}}$ largest observed values of Z_{ij}, $i < j$, to obtain by simulation the $r(r-1)/2$ possible observed significance levels.

The K-S (Kruskal-Scheffé) method is also sometimes used in this situation (see, e.g., Miller, 1966, p. 166) and consists of replacing $z_\alpha/(r(r-1))$ in (3.1) with $h_\alpha^{1/2} = (\chi^2_{\alpha;r-1})^{1/2}$, where $\chi^2_{\alpha;r-1}$ is the upper $\alpha^{\underline{th}}$ point of χ^2 with r-1 degrees of freedom. The comparison of the critical constants in Table 3 shows that this is even more conservative than D-B.

3. Comparison of D-B and K-S Critical Constants

r	$z_{.05}/(r(r-1))$	$(\chi^2_{.05;r-1})^{1/2}$	$z_{.01}/(r(r-1))$	$(\chi^2_{.01;r-1})^{1/2}$
3	2.39	2.79	2.94	3.36
4	2.50	3.08	3.02	3.65
5	2.58	3.33	3.09	3.89
6	2.64	3.55	3.15	4.10
7	2.69	3.75	3.19	4.30
8	2.74	3.94	3.23	4.48
9	2.77	4.11	3.26	4.66
10	2.81	4.28	3.29	4.82

If the data is discrete, the D-B method can be modified as in Dunn (1964) and the random assignment done on average ranks. Thus ties present no problems in this approach.

For all practical purposes the exact D-B (use of the $d_{(i)}$ and simulation) seems the best method to use.

Appendix

```
C EXAMPLE OF MAIN PROGRAM FOR SEQUENTIAL RANDOM ALLOCATIONS
C NR IS THE NUMBER OF GROUPS, NSIM THE NUMBER OF SIMULATIONS, K'S THE GROUP
C SIZES, X'S THE NUMBERS TO BE ALLOCATED
      DIMENSION X(1000),Z(20,100),K(20)
  100 FORMAT ( )
   99 READ 100 ,NR,NSIM
      IF (NR.EQ. 0) GO TO 101
      READ 100,(K(J),J=1,NR)
      KK=0
      DO 1 J=1,NR
    1 KK=KK+K(J)
      DO 203 I=1,KK
  203 X(I)=I
      DO 2 II=1,NSIM
      CALL RANDM(NN,K,KK,X,Z)
    2 CONTINUE
      GO TO 99
  101 STOP
      END
$FOR,IS ,SUB1
      SUBROUTINE RANDM(NR,K,KK,X,Z)
C NR NUMBER OF GROUPS, K ARRAY OF GROUP SIZES, KK NUMBER OF ELEMENTS, X ARRAY
C OF KK ELEMENTS, Z RANDOM ALLOCATION OF X
      DIMENSION X(1000),Z(20,100),NSC(20),Q(20),QC(20),U(1000),K(20)
      DO 3 I2=1,NR
    3 NSC(I2)=0
      DO 333 I2=1,KK
  333 U(I2)=RANUN(X)
      DO 4 I3=1,NR
      QC(I3)=0.
    4 Q(I3)=K(I3)
      MAXI=NR-1
      DO 5 II=1,MAXI
      DO 5 III=1,II
    5 QC(II)=QC(II)+Q(III)
      DO 6 I4=1,KK
      U(I4)=(KK-I4+1.)*U(I4)
      IF (U(I4).LE.Q(1)) GO TO 61
      DO 7 I5=2,MAXI
    7 IF (U(I4).LE.QC(I5)) GO TO 62
      IF (U(I4).GT.QC(NR-1)) INDEX=NR
      GO TO 64
   61 INDEX=1
      GO TO 64
   62 INDEX=I5
   64 NSC(INDEX)=NSC(INDEX)+1
      NN=NSC(INDEX)
      Z(INDEX,NN)=X(I4)
C UPDATE
      DO 8 I6=1,NR
      QC(I6)=0.
    8 Q(I6)= K(I6)-NSC(I6)
      DO 9 II=1,MAXI
      DO 9 III=1,II
    9 QC(II)=QC(II)+Q(III)
    6 CONTINUE
      RETURN
      END
```

```
C D-R BY SIMULATION
C NR-IS THE NUMBER OF GROUPS, NSIM THE NUMBER OF SIMULATIONS, ZAL IS THE POINT
C FOR WHICH THE PROBABILITY OF THE MAXIMUM OF ALL THE ABSOLUTE VALUES OF
C STANDARDIZED RANK AVERAGE DIFFERENCES EQUALLING OR EXCEEDING IT IS TO BE
C CALCULATED, K'S ARE THE GROUP SIZES
      DIMENSION IFLAG(20,20),R(20)
      DIMENSION X(1000),Z(20,100),K(20)
  100 FORMAT ( )
   99 READ 100 ,NR,NSIM
      IF (NR.EQ. 0) GO TO 101
      MAXI=NR-1
      READ 100,ZAL
      READ 100,(K(J),J=1,NR)
      KK=0
      DO 1 J=1,NR
    1 KK=KK+K(J)
      DO 203 I=1,KK
  203 X(I)=I
      CON=KK*(KK+1)*ZAL**2
      COUNT=0.
      DO 2 II=1,NSIM
      CALL RANDM(NR,K,KK,X,Z)
      DO 20 J1=1,NR
      R(J1)=0.
      NUPP=K(J1)
      DO 20 J2=1,NUPP
   20 R(J1)=R(J1)+Z(J1,J2)
      DO 21 J1=1,MAXI
      LLIM=J1+1
      DO 21 J2=LLIM,NR
      IFLAG(J1,J2)=0
   21 IF (12*(K(J2)*R(J1)-K(J1)*R(J2))**2.GE.K(J1)*K(J2)*CON*(K(J1)+K(J2
     1))) IFLAG(J1,J2)=1
      IPROD=0
      DO 22 J1=1,MAXI
      LLIM=J1+1
      DO 22 J2=LLIM,NR
   22 IPROD=IPROD+IFLAG(J1,J2)
      IF (IPROD.GE.1) COUNT=COUNT+1
    2 CONTINUE
      PR=COUNT/NSIM
      PRINT 100,(K(I),I=1,NR)
      PRINT 100,ZAL,NSIM,PR
      GO TO 99
  101 STOP
      END
```

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